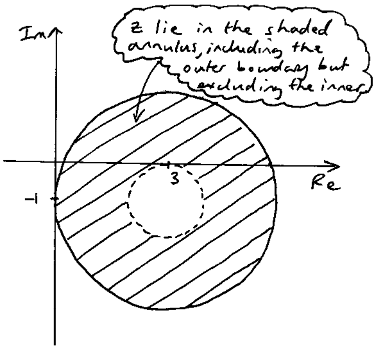
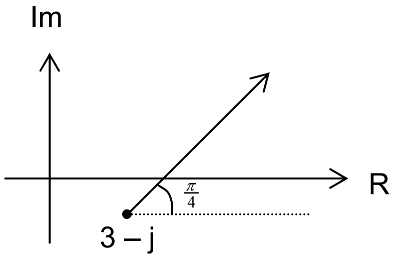
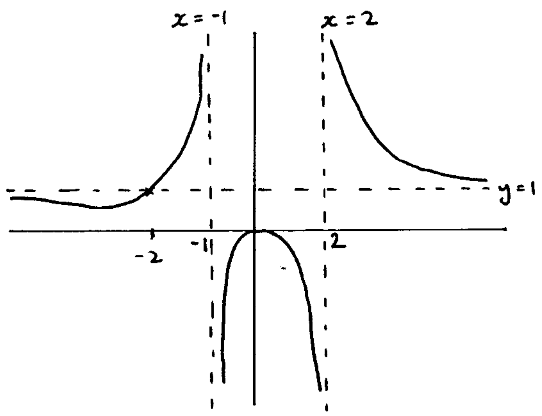
Mark Scheme 4755
June 2006

Qu	Answer	Mark	Comment
Section A			
1 (i)	Reflection in the x-axis.	B1 **[1]**	
1(ii)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1 **[1]**	
1(iii)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	M1 A1 c.a.o. **[2]**	Multiplication of their matrices in the correct order or B2 for correct matrix without working
2	$(x+2)(Ax^2+Bx+C)+D$ $= Ax^3+Bx^2+Cx+2Ax^2+2Bx+2C+D$ $= Ax^3+(2A+B)x^2+(2B+C)x+2C+D$ $\Rightarrow A=2,\ B=-7,\ C=15,\ D=-32$	M1 B1 B1 F1 F1 **OR** B5 **[5]**	Valid method to find all coefficients For A = 2 For D = -32 F1 for each of B and C For all correct
3(i)	$\alpha+\beta+\gamma=-4$ $\alpha\beta+\beta\gamma+\alpha\gamma=-3$ $\alpha\beta\gamma=-1$	B1 B1 B1 **[3]**	
3(ii)	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma)$ $=16+6=22$	M1 A1 E1 **[3]**	Attempt to use $(\alpha+\beta+\gamma)^2$ Correct Result shown
4 (i)	Argand diagram with solid circle, centre 3 – j, radius 3, with values of z on and within the circle clearly indicated as satisfying the inequality.	B1 B1 B1 **[3]**	Circle, radius 3, shown on diagram Circle centred on 3 - j Solution set indicated (solid circle with region inside)
4(ii)		B1 B1 **[2]**	 Hole, radius 1, shown on diagram Boundaries dealt with correctly

Qu	Answer	Mark	Comment
Section A (continued)			
4(iii)		B1 B1 B1 [3]	Line through their 3 – j Half line $\frac{\pi}{4}$ to real axis
5(i)	$\begin{pmatrix}-1 & 2\\-3 & 4\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$	B1	
	$\mathbf{S}^{-1}=\frac{1}{2}\begin{pmatrix}4 & -2\\3 & -1\end{pmatrix}$	M1, A1	Attempt to divide by determinant and manipulate contents Correct
	$\frac{1}{2}\begin{pmatrix}4 & -2\\3 & -1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$	E1 [4]	
5(ii)	$\mathbf{T}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$		
	$\Rightarrow \mathbf{T}^{-1}\mathbf{T}\begin{pmatrix}x\\y\end{pmatrix}=\mathbf{T}^{-1}\begin{pmatrix}x\\y\end{pmatrix}$	M1	Pre-multiply by $\mathbf{T}^{-1}$
	$\Rightarrow \begin{pmatrix}x\\y\end{pmatrix}=\mathbf{T}^{-1}\begin{pmatrix}x\\y\end{pmatrix}$	A1 [2]	Invariance shown
6	$3+6+12+........+3\times 2^{n-1}=3\left(2^{n}-1\right)$ n = 1, LHS = 3, RHS = 3	B1	
	Assume true for n = k Next term is $3\times 2^{k+1-1}=3\times 2^{k}$ Add to both sides	E1 B1	Assuming true for k $(k+1)^{\text{th}}$ term.
	$\text{RHS}=3\left(2^{k}-1\right)+3\times 2^{k}$	M1	Add to both sides
	$=3\left(2^{k}-1+2^{k}\right)$ $=3\left(2\times 2^{k}-1\right)$ $=3\left(2^{k+1}-1\right)$	A1	Working must be valid
	But this is the given result with k + 1 replacing k. Therefore if it is true for k it is true for k + 1. Since it is true for k = 1, it is true for all positive integers n.	E1 E1 [7]	Dependent on previous A1and E1 Dependent on B1 and previous E1
			Section A Total: 36

Section B

7(i)	$x=2$, $x=-1$ and $y=1$	B1 B1B1 **[3]**	One mark for each
7(ii) **(*A*)** **(*B*)**	Large positive *x*, $y \to 1^+$ (from above) (e.g. consider $x=100$) Large negative *x*, $y \to 1^-$ (from below) (e.g. consider $x=-100$)	M1 B1 B1 **[3]**	Evidence of method needed for M1
7(iii)	Curve 3 branches Correct approaches to horizontal asymptote Asymptotes marked Through origin 	B1 B1 B1 B1 **[4]**	With correct approaches to vertical asymptotes Consistent with their (i) and (ii) Equations or values at axes clear
7(iv)	$x<-1,\ x>2$	B1B1, B1, **[3]**	s.c. 1 for inclusive inequalities Final B1 for all correct with no other solutions

8(i)	$(2+j)^2 = 3+4j$	B1	
	$(2+j)^3 = 2+11j$	B1	
	Substituting into $2x^3 - 11x^2 + 22x - 15$:	M1	Attempt at substitution
	$2(2+11j) - 11(3+4j) + 22(2+j) - 15$	A1	Correctly substituted
	$= 4 + 22j - 33 - 44j + 44 + 22j - 15$ $= 0$ So 2 + j is a root.	A1 **[5]**	Correctly cancelled (Or other valid methods)
8(ii)	2 - j	B1 **[1]**	
8(iii)	$(x-(2+j))(x-(2-j))$ $= (x-2-j)(x-2+j)$	M1	Use of factor theorem
	$= x^2 - 2x + jx - 2x + 4 - 2j - jx + 2j + 1$ $= x^2 - 4x + 5$	A1	
	$(x^2 - 4x + 5)(ax + b) = 2x^3 - 11x^2 + 22x - 15$ $(x^2 - 4x + 5)(2x - 3) = 2x^3 - 11x^2 + 22x - 15$	M1	Comparing coefficients or long division
	$(2x-3) = 0 \Rightarrow x = \frac{3}{2}$	A1 **[4]**	Correct third root
	OR		
	Sum of roots = $\frac{11}{2}$ **or** product of roots = $\frac{15}{2}$	M1 A1	
	leading to $\alpha + 2 + j + 2 - j = \frac{11}{2}$	M1	
	$\Rightarrow \alpha = \frac{3}{2}$	A1 **[4]**	
	or $\alpha(2+j)(2-j) = \frac{15}{2}$	M1 A1	
	$\Rightarrow 5\alpha = \frac{15}{2} \Rightarrow \alpha = \frac{3}{2}$	M1 A1 **[4]**	(Or other valid methods)

9(i)	$r(r+1)(r+2)-(r-1)r(r+1)$ $\equiv (r^2+r)(r+2)-r^3-r$ $\equiv r^3+2r^2+r^2+2r-r^3+r$ $\equiv 3r^2+3r \equiv 3r(r+1)$	M1 E1 **[2]**	Accept '=' in place of '≡' throughout working Clearly shown
9(ii)	$\sum_{r=1}^{n} r(r+1)$ $=\frac{1}{3}\sum_{r=1}^{n}[r(r+1)(r+2)-(r-1)r(r+1)]$ $=\frac{1}{3}[(1\times 2\times 3-0\times 1\times 2)+(2\times 3\times 4-1\times 2\times 3)+$ $(3\times 4\times 5-2\times 3\times 4)+......$ $+(n(n+1)(n+2)-(n-1)n(n+1))]$ $=\frac{1}{3}n(n+1)(n+2)$ or equivalent	M1 M1 A2 M1 A1 **[6]**	Using identity from (i) Writing out terms in full At least 3 terms correct (minus 1 each error to minimum of 0) Attempt at eliminating terms (telescoping) Correct result
9(iii)	$\sum_{r=1}^{n} r(r+1)=\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} r$ $=\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)$ $=\frac{1}{6}n(n+1)[(2n+1)+3]$ $=\frac{1}{6}n(n+1)(2n+4)$ $=\frac{1}{3}n(n+1)(n+2)$ or equivalent	B1 B1 M1 A1 E1 **[5]**	Use of standard sums (1 mark each) Attempt to combine Correctly simplified to match result from (ii)
			Section B Total: 36
			Total: 72